

Unit-2. Electrostatics - Short Solutions

Part A: Definitions (1-2 marks)

Electric Field (E)

Force per unit positive charge at a point.

$$E = F/q_0 = kQ/r^2$$

Unit: N/C or V/m

Electric Potential (V)

Work done to bring unit positive charge from infinity to a point.

$$V = W/q = kQ/r$$

Unit: Volt (V) or J/C

Electric Potential Difference (ΔV)

Work done to move unit charge from one point to another.

$$\Delta V = W/q = V_2 - V_1$$

Unit: Volt (V)

Electric Flux (Φ)

Number of electric field lines passing through a surface.

$$\Phi = E \cdot A = EA \cos \theta$$

Unit: N·m²/C or V·m

Capacitor

Device that stores electric charge and energy. Two conducting plates separated by dielectric.

Capacitance (C)

Ability to store charge. Ratio of charge to potential difference.

$$C = Q/V$$

$$C \text{ (parallel plate)} = \epsilon_0 \epsilon_r A/d$$

Unit: Farad (F)

Part B: Detailed Answers (2-3 marks)

(1) Coulomb's Law

Electric force between two stationary point charges is directly proportional to product of charges and inversely proportional to square of distance.

$$F = kq_1q_2/r^2$$
$$k = 9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

Nature:

- Like charges: Repulsive
- Unlike charges: Attractive

(2) Characteristics of Electric Field Lines

1. **Start/End:** Start from +ve, end at -ve charges
2. **Direction:** Tangent shows field direction
3. **No intersection:** Lines never cross
4. **Density:** Close lines = strong field, far lines = weak field
5. **Uniform field:** Parallel and equidistant
6. **Imaginary:** Lines are imaginary, field is real
7. **Perpendicular:** Always perpendicular to conductor surface
8. **Open curves:** Never form closed loops

(3) Parallel Plate Capacitor

Construction: Two parallel conducting plates of area A separated by distance d with dielectric between.

Capacitance:

$$C = \epsilon_0 \epsilon_r A/d = \epsilon_0 K A/d$$

Factors affecting C:

- $C \propto A$ (area)
- $C \propto 1/d$ (distance)
- $C \propto K$ (dielectric constant)

Applications: Energy storage, filtering, timing circuits, coupling/decoupling

(4) Series Connection of Capacitors

Characteristics:

- Same charge: $Q_1 = Q_2 = Q_3 = Q$

- Different voltages: $V = V_1 + V_2 + V_3$

Formula:

$$1/C_s = 1/C_1 + 1/C_2 + 1/C_3$$

For 2 capacitors: $C_s = C_1 C_2 / (C_1 + C_2)$

For n equal: $C_s = C/n$

Result: $C_s <$ smallest capacitance

(5) Parallel Connection of Capacitors

Characteristics:

- Same voltage: $V_1 = V_2 = V_3 = V$
- Different charges: $Q = Q_1 + Q_2 + Q_3$

Formula:

$$C_p = C_1 + C_2 + C_3$$

For n equal: $C_p = nC$

Result: $C_p >$ largest capacitance

(6) Effect of Dielectric on Capacitance

When dielectric (K) inserted between plates:

Capacitance increases:

$$C = KC_0$$

Why it increases:

- Dielectric molecules polarize
- Reduces electric field: $E = E_0/K$
- Reduces voltage: $V = V_0/K$
- Same charge, lower voltage \rightarrow higher capacitance

Other effects:

- Increases breakdown voltage
- Provides mechanical support
- Makes capacitor compact

Part C: Numericals (3 marks)

(1) Coulomb force

Given: $q_1 = 20 \mu\text{C}$, $q_2 = 10 \mu\text{C}$, $r = 0.02 \text{ m}$, $k = 9 \times 10^9$

$$\begin{aligned}F &= kq_1q_2/r^2 \\F &= (9 \times 10^9)(20 \times 10^{-6})(10 \times 10^{-6})/(0.02)^2 \\F &= 1800 \times 10^{-3}/4 \times 10^{-4} = 4500 \text{ N}\end{aligned}$$

Answer: 4500 N (repulsive)

(2) Potential difference

Given: $W = 1600 \text{ J}$, $q = 25 \text{ C}$

$$V = W/q = 1600/25 = 64 \text{ V}$$

Answer: 64 V

(3) Capacitance

Given: $Q = 60 \mu\text{C}$, $V = 12 \text{ V}$

$$C = Q/V = 60 \times 10^{-6}/12 = 5 \times 10^{-6} \text{ F} = 5 \mu\text{F}$$

Answer: 5 μF

(4) Series and Parallel ($3 \times 10 \mu\text{F}$)

Series:

$$C_s = C/n = 10/3 = 3.33 \mu\text{F}$$

Parallel:

$$C_p = nC = 3 \times 10 = 30 \mu\text{F}$$

Answer: Series = 3.33 μF , Parallel = 30 μF

(5) Capacitance of small capacitor

Given: $A = 10 \text{ mm}^2 = 10^{-5} \text{ m}^2$, $d = 1 \text{ mm} = 10^{-3} \text{ m}$

$$\begin{aligned}C &= \epsilon_0 A/d = (8.85 \times 10^{-12})(10^{-5})/(10^{-3}) \\C &= 8.85 \times 10^{-14} \text{ F} = 0.0885 \text{ pF}\end{aligned}$$

Answer: $8.85 \times 10^{-14} \text{ F}$

(6) Area for 1F capacitor

Given: $C = 1 \text{ F}$, $d = 1 \text{ mm} = 10^{-3} \text{ m}$

$$A = Cd/\epsilon_0 = (1)(10^{-3})/(8.85 \times 10^{-12})$$
$$A = 1.13 \times 10^8 \text{ m}^2 = 113 \text{ km}^2$$

Answer: $1.13 \times 10^8 \text{ m}^2$ (113 km²) - Shows 1F is huge!

(7) Mixed Circuit

Example: $(C_1 \parallel C_2)$ in series with C_3

Given: $C_1 = 10 \mu\text{F}$, $C_2 = 20 \mu\text{F}$, $C_3 = 30 \mu\text{F}$

Step 1: Parallel

$$C_{12} = C_1 + C_2 = 10 + 20 = 30 \mu\text{F}$$

Step 2: Series with C_3

$$1/C_t = 1/30 + 1/30 = 2/30$$
$$C_t = 15 \mu\text{F}$$

Answer: 15 μF

Quick Reference

Formulas

Coulomb's Law: $F = kq_1q_2/r^2$, $k = 9 \times 10^9$

Electric Field: $E = F/q = kQ/r^2$

Potential: $V = W/q = kQ/r$

Flux: $\Phi = EA \cos \theta$

Capacitance: $C = Q/V = \epsilon_0 \epsilon_r A/d$

Series: $1/C_s = 1/C_1 + 1/C_2 + 1/C_3$

Parallel: $C_p = C_1 + C_2 + C_3$

With dielectric: $C = KC_0$

Energy: $U = \frac{1}{2}QV = \frac{1}{2}CV^2 = Q^2/(2C)$

Constants

$k = 9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$
 $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$
 $e = 1.6 \times 10^{-19} \text{ C}$
1 $\mu\text{F} = 10^{-6} \text{ F}$
1 $\text{nF} = 10^{-9} \text{ F}$
1 $\text{pF} = 10^{-12} \text{ F}$

Short Solutions - Unit 2